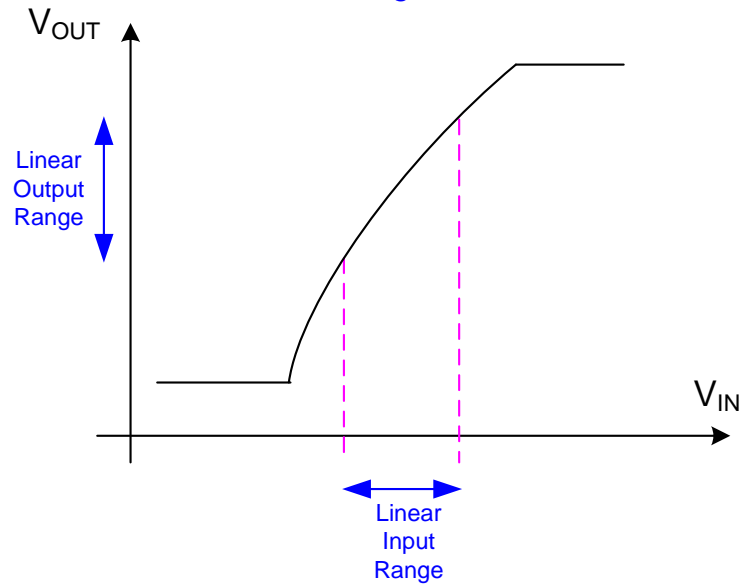
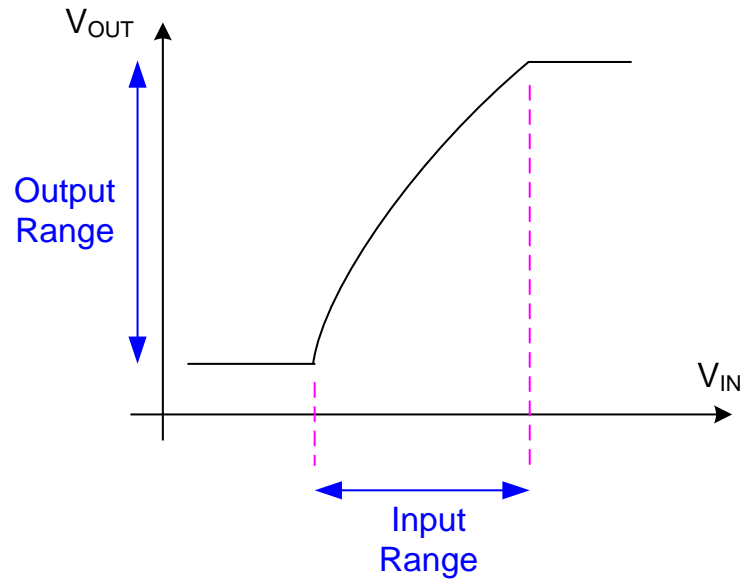


# EE 435

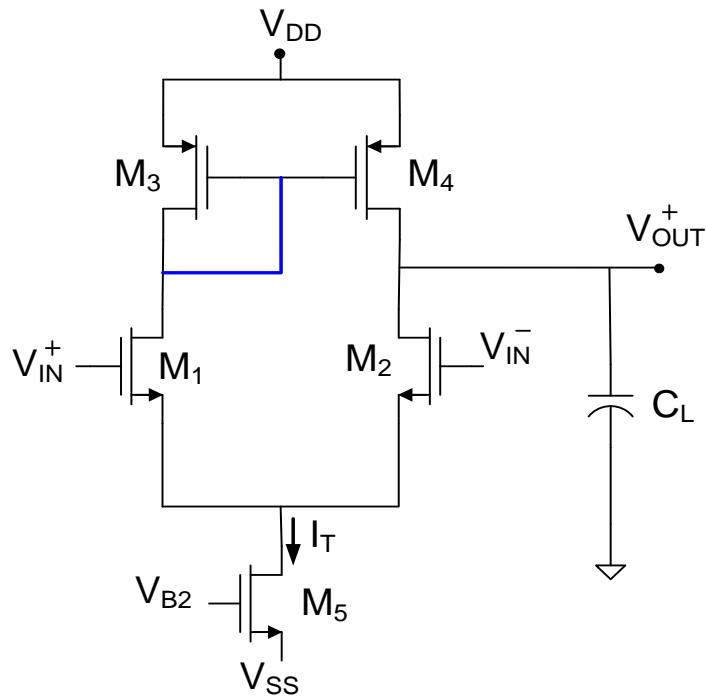
## Lecture 21

Linearity of Bipolar and MOS Differential Pairs  
Linearity of Common Source Amplifier  
Offset Voltages

# Signal Swing and Linearity

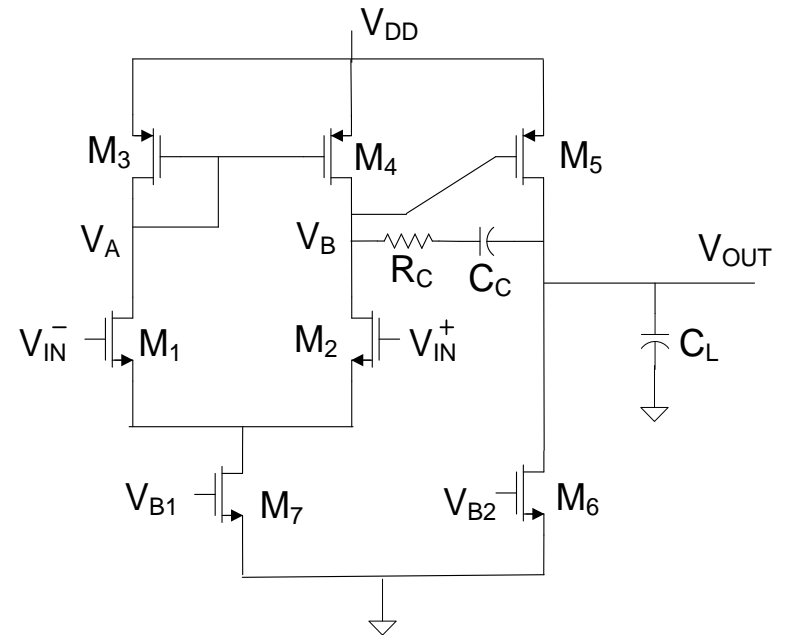


# Linearity of Amplifiers



Single-Stage

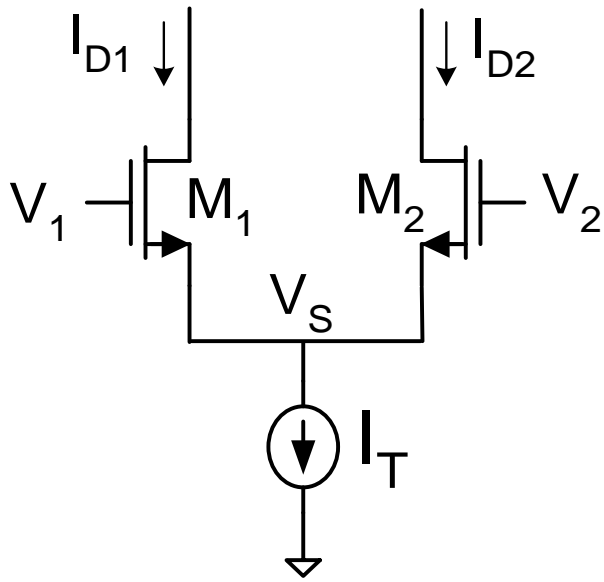
Linearity of differential pair of major concern



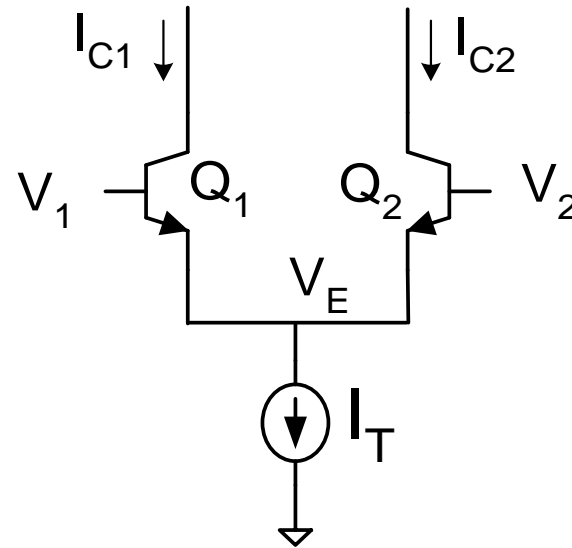
Two-Stage

Linearity of common-source amplifier is of major concern (since signals so small at output of differential pair)

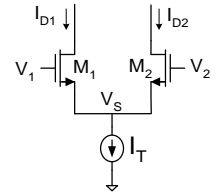
# Differential Input Pairs



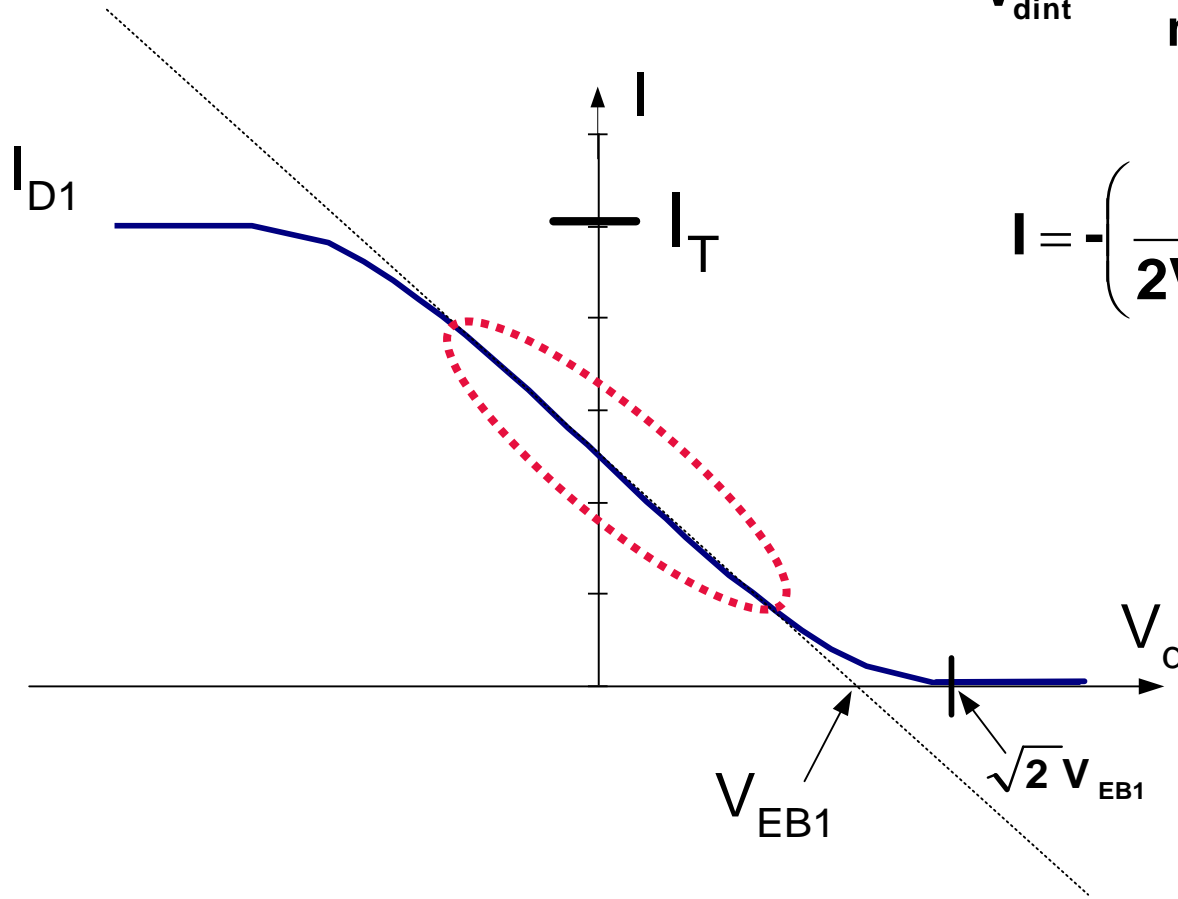
MOS Differential Pair



Bipolar Differential Pair



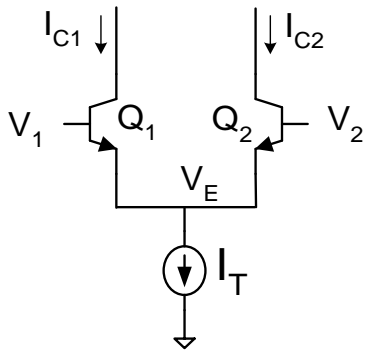
# How linear is the amplifier ?



$$V_{dint} = -\frac{h}{m} = -\frac{I_T}{2m} = V_{EB1}$$

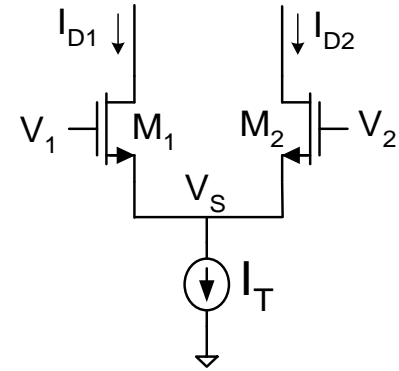
$$I = -\left(\frac{I_T}{2V_{EB1}}\right) V_d + \frac{I_T}{2}$$

# Comparison of Distortion in BJT and MOSFET Pairs



$$V_d = V_m \sin(\omega t)$$

$$\text{THD} = -20 \log \left( 48 \left( \frac{V_t}{V_m} \right)^2 - 3 \right)$$

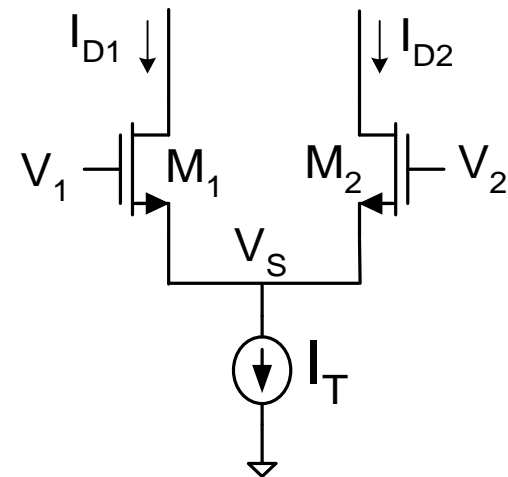
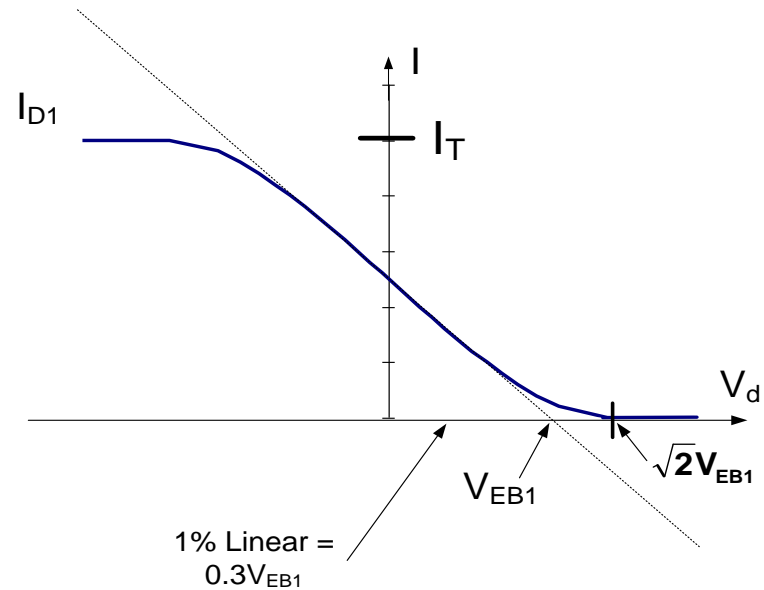
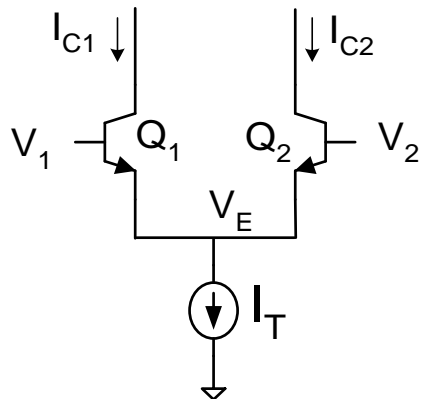
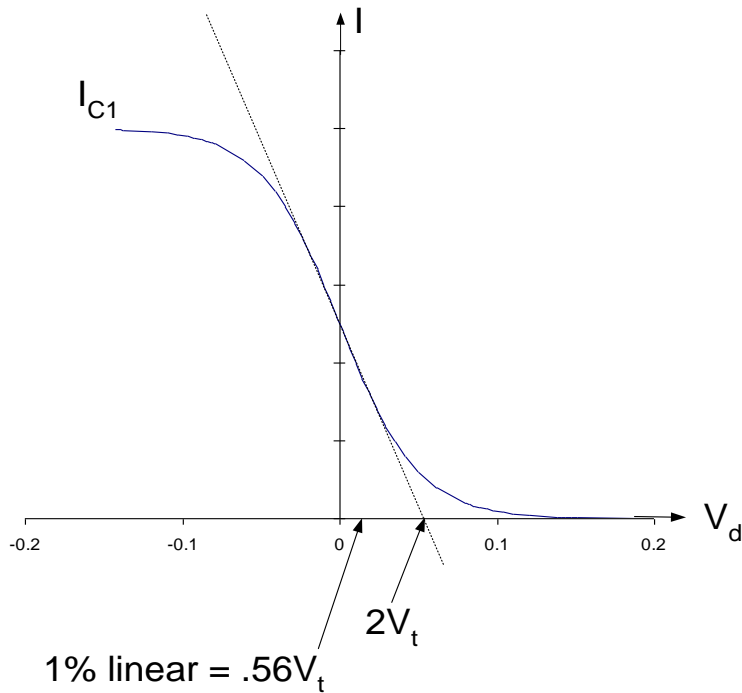


$$\text{THD} = -20 \log \left( 32 \left( \frac{V_{EB1}}{V_m} \right)^2 - 3 \right)$$

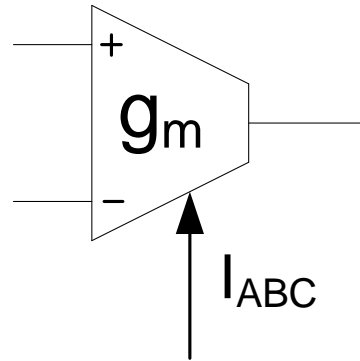
$V_m / V_t$	THD (dB)	$V_m / V_{EB1}$	THD (dB)
2.5	-13.4049	2.5	-6.52672
1	-33.0643	1	-29.248
0.5	-45.5292	0.5	-41.9382
0.25	-57.6732	0.25	-54.1344
0.1	-73.6194	0.1	-70.0949
0.05	-85.6647	0.05	-82.1422
0.025	-97.7069	0.025	-94.1849
0.01	-113.625	0.01	-110.103

Review from last lecture

# Linearity and Signal Swing Comparison of Bipolar/MOS Differential Pair



# Applications as a programmable OTA



The current-dependence of the  $g_m$  of the differential pair (single transistor) is often used to program the transconductance of an OTA with the tail bias current  $I_{ABC}$

MOS

$$g_m = \sqrt{I_{ABC}} \sqrt{\mu C_{OX} \frac{W}{L}}$$

Two decade change in current for every decade change in  $g_m$

$$g_m = \mu C_{OX} \frac{W}{L} V_{EB}$$

One decade decrease in signal swing for every decade decrease in  $g_m$

Limited  $g_m$  adjustment possibility

BJT

$$g_m = \frac{I_{ABC}}{2V_t}$$

One decade change in current for every decade change in  $g_m$

No change in signal swing when  $g_m$  is changed

Large  $g_m$  adjustment possible

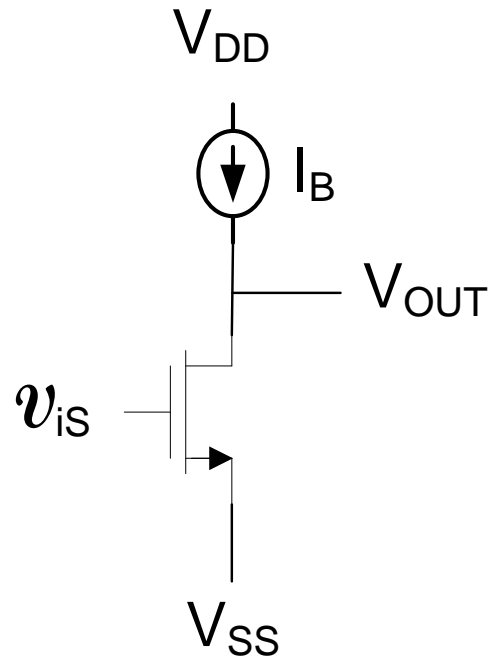


# Signal Swing and Linearity Summary

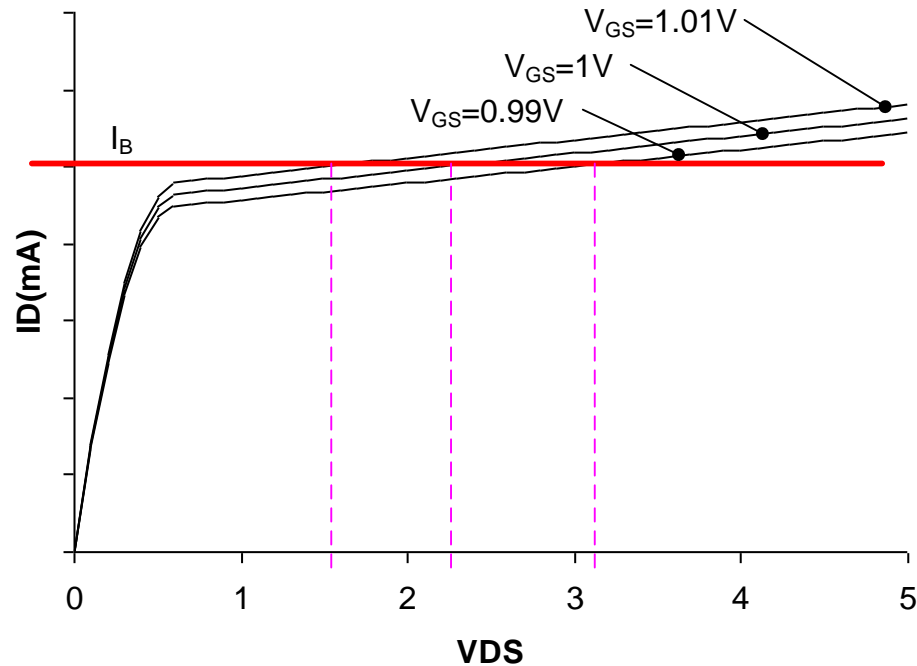
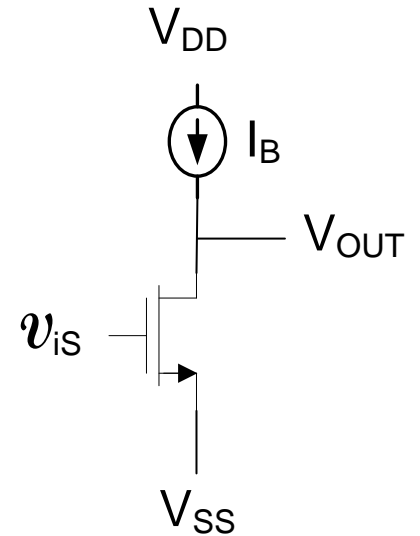
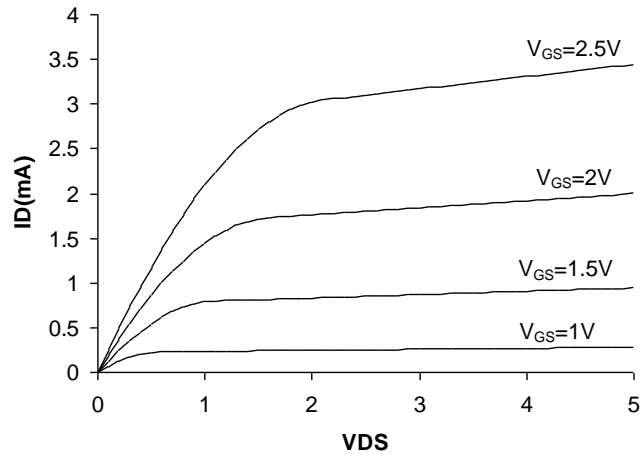
- Signal swing of MOSFET can be rather large if  $V_{EB}$  is large but this limits gain
- Signal swing of MOSFET degrades significantly if  $V_{EB}$  is changed for fixed  $W/L$
- Bipolar swing is very small but independent of  $g_m$
- Multiple-decade adjustment of bipolar  $g_m$  is practical
- Even though bipolar input swing is small, since gain is often very large, this small swing does usually not limit performance in feedback applications

# Linearity of Common-Source Amplifier

For convenience, will consider situation where current source biasing  $I_B$  is ideal



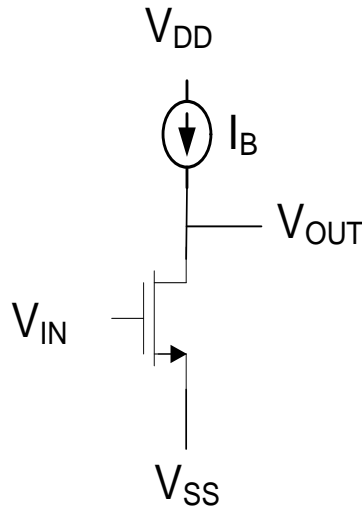
# Linearity of Common-Source Amplifier



# Linearity of Common-Source Amplifier

$$V_{IN} = V_{INQ} + v_{iS}$$

$V_{INQ}$ : Quiescent Input  
 $v_{iS}$ : Signal Input



$$V_{OUT} = V_{OQ} + v_{oS}$$

$V_{OQ}$ : Quiescent Output  
 $v_{oS}$ : Signal Output

$$I_B = \frac{\mu C_{OX} W}{2L} (V_{IN} - V_{SS} - V_{TH})^2 (1 + \lambda [V_{OUT} - V_{SS}])$$

$V_{EB} = V_{INQ} - V_{SS} - V_{TH}$  strictly for notational convenience define  $\beta = \frac{\mu C_{OX} W}{2L}$

$$I_B = \beta (v_{iS} - V_{EB})^2 (1 + \lambda [v_{oS} + V_{OQ} - V_{SS}])$$

$$v_{oS} = V_{SS} - V_{OQ} - \frac{\left( \frac{I_B}{\beta V_{EB}^2 \left( 1 - \frac{v_{iS}}{V_{EB}} \right)^2} \right)^{-1}}{\lambda}$$

# Linearity of Common-Source Amplifier

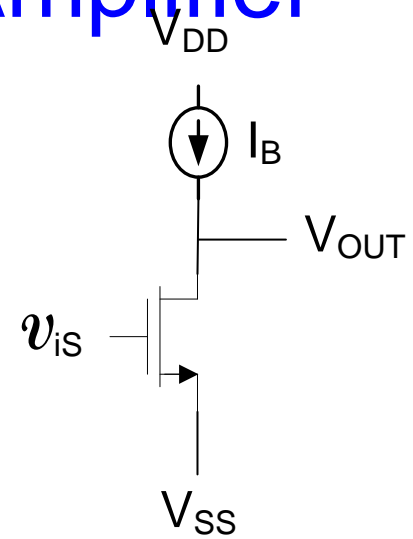
$$v_{OS} = V_{SS} - V_{OQ} - \frac{\left( \frac{I_B}{\beta V_{EB}^2 \left( 1 - \frac{v_{iS}}{V_{EB}} \right)^2} \right)^{-1}}{\lambda}$$

Recall for x small  $\frac{1}{1+x} \approx 1-x$

$$v_{OS} \approx V_{SS} - V_{OQ} - \frac{\left( \frac{I_B \left( 1 + \frac{v_{iS}}{V_{EB}} \right)^2}{\beta V_{EB}^2} \right)^{-1}}{\lambda}$$

$$v_{OS} \approx V_{SS} - V_{OQ} - \frac{I_B}{\lambda \beta V_{EB}^2} \left( 1 + 2 \frac{v_{iS}}{V_{EB}} + \left( \frac{v_{iS}}{V_{EB}} \right)^2 \right) - \frac{1}{\lambda}$$

$$v_{OS} \approx \left[ V_{SS} - V_{OQ} + \frac{1}{\lambda} \left( \frac{I_B}{\beta V_{EB}^2} - 1 \right) \right] - \frac{I_B}{\lambda \beta V_{EB}^2} \left( 2 \frac{v_{iS}}{V_{EB}} + \left( \frac{v_{iS}}{V_{EB}} \right)^2 \right)$$



# Linearity of Common-Source Amplifier

$$v_{OS} \cong \left[ V_{SS} - V_{OQ} + \frac{1}{\lambda} \left( \frac{I_B}{\beta V_{EB}^2} - 1 \right) \right] - \frac{I_B}{\lambda \beta V_{EB}^2} \left( 2 \frac{v_{iS}}{V_{EB}} + \left( \frac{v_{iS}}{V_{EB}} \right)^2 \right)$$

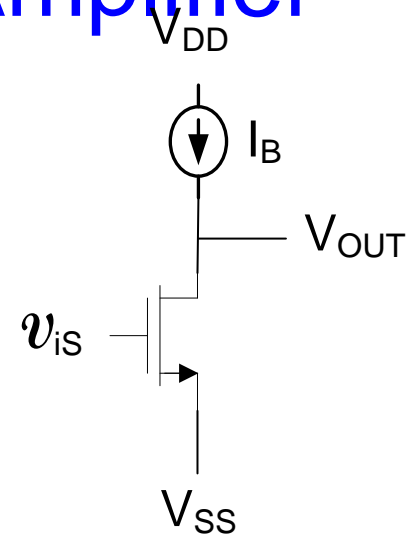
$$\text{but } \left[ V_{SS} - V_{OQ} + \frac{1}{\lambda} \left( \frac{I_B}{\beta V_{EB}^2} - 1 \right) \right] \cong 0$$

$$I_B \cong \beta (V_{EB})^2$$

Thus

$$v_{OS} \cong - \left( 2 \frac{v_{iS}}{\lambda V_{EB}} + \frac{1}{\lambda} \left( \frac{v_{iS}}{V_{EB}} \right)^2 \right)$$

$$v_{OS} \cong - \frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

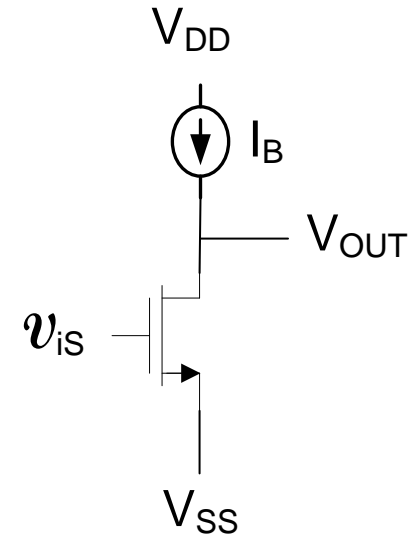
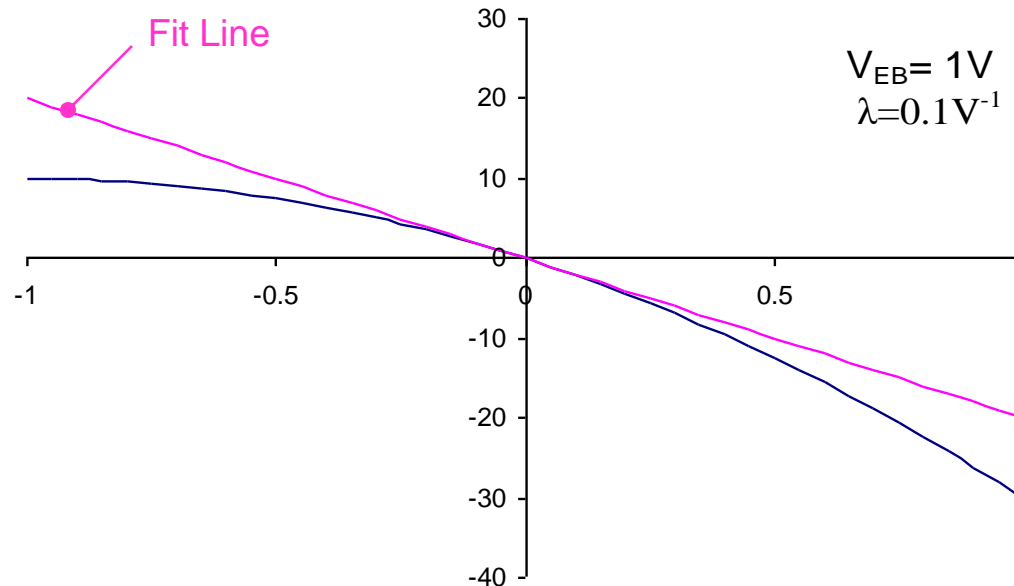


Is this a linear or nonlinear relationship?

# Linearity of Common-Source Amplifier

$$v_{OS} \cong \frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



when  $v_{iS} = -V_{EB}$  (the minimum value of  $v_{iS}$  to maintain saturation operation)

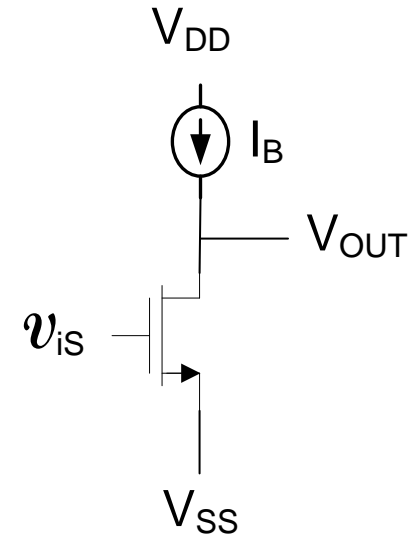
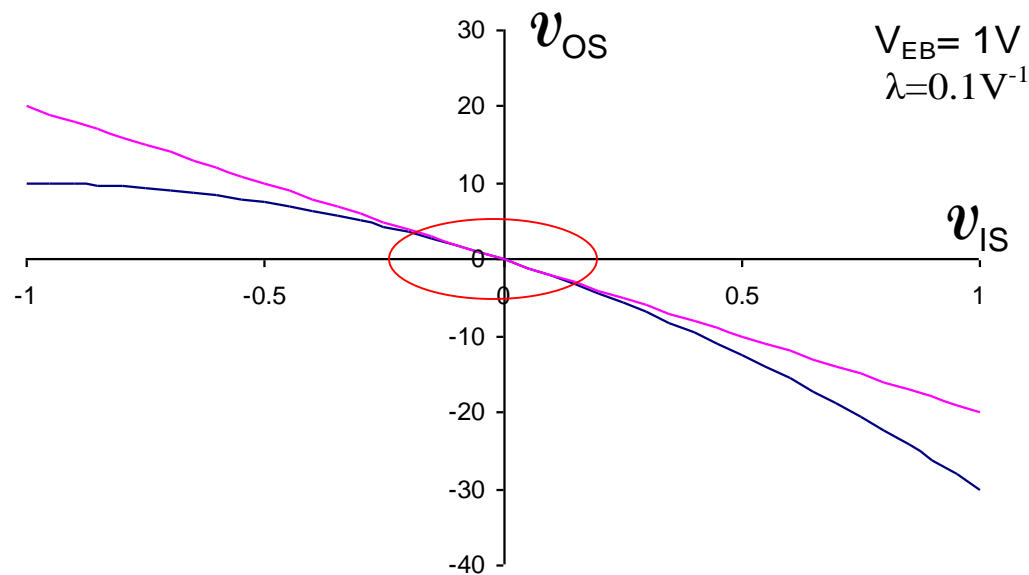
the error in  $V_{OS}$  will be  $V_{EB}/2$  which is -50% !

Is this a linear or nonlinear relationship?

# Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



Note this is a reasonably high gain amplifier and could be larger for smaller  $V_{EB}$

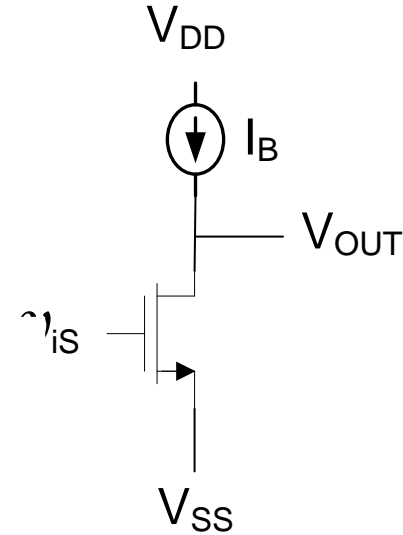
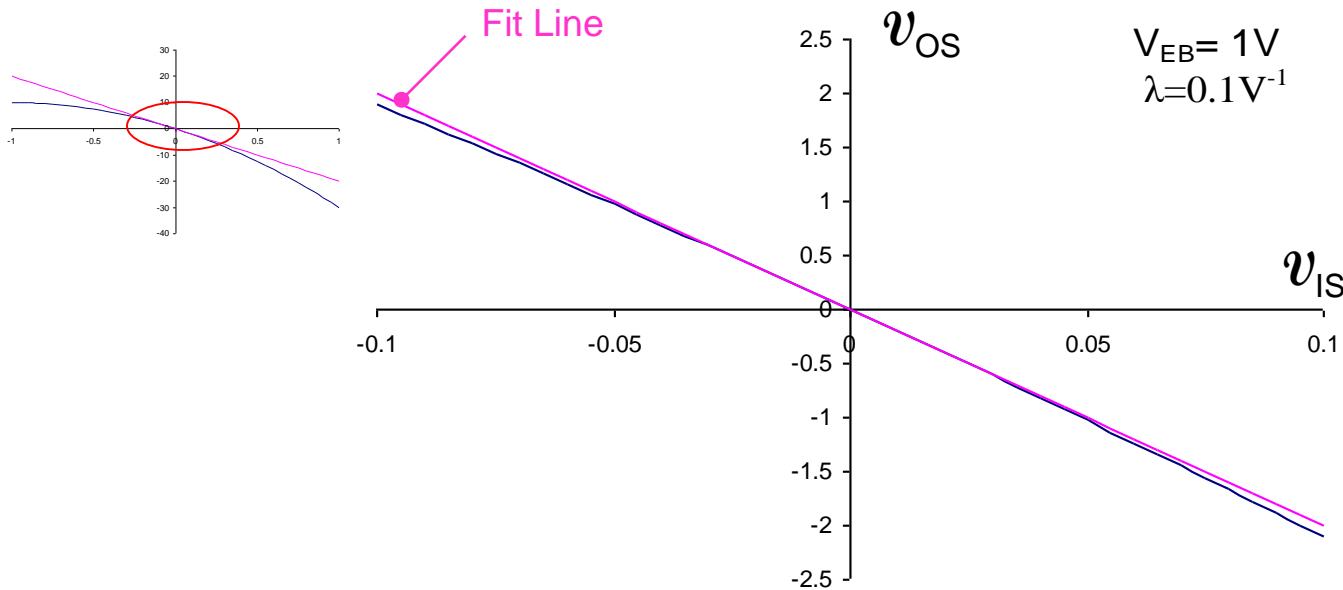
Over what output voltage range are we interested?



# Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



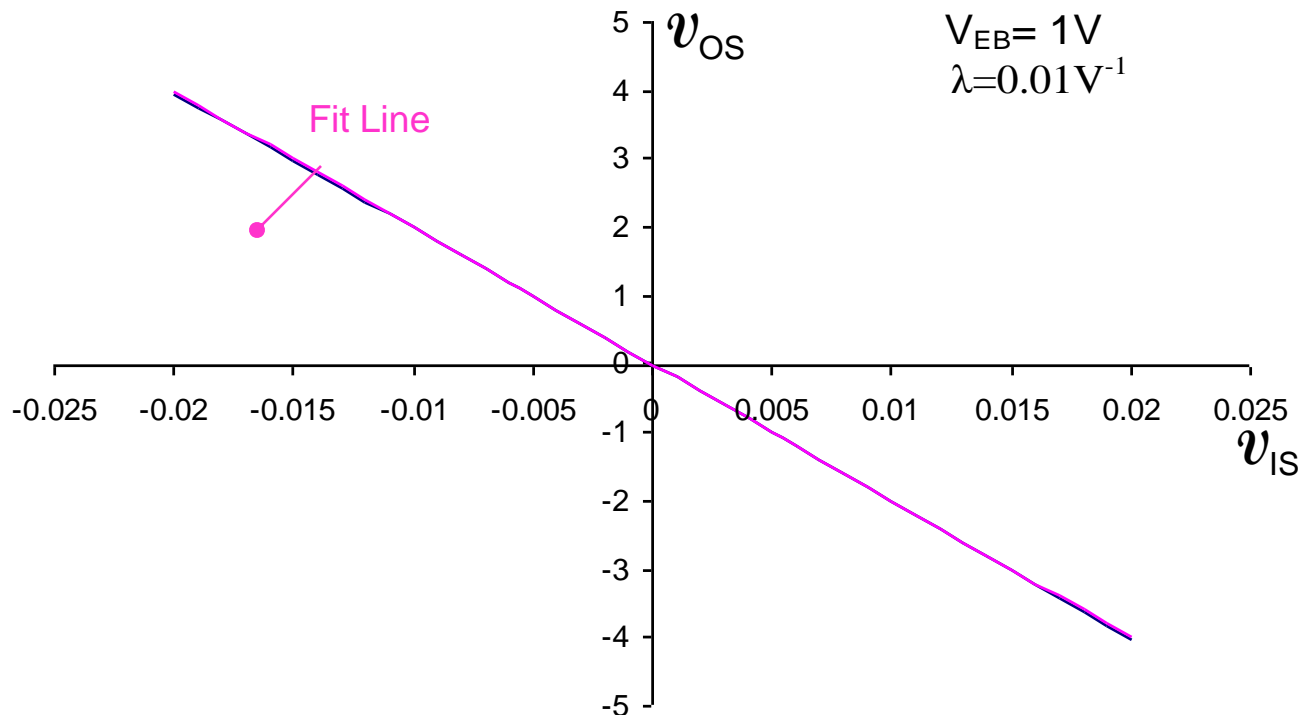
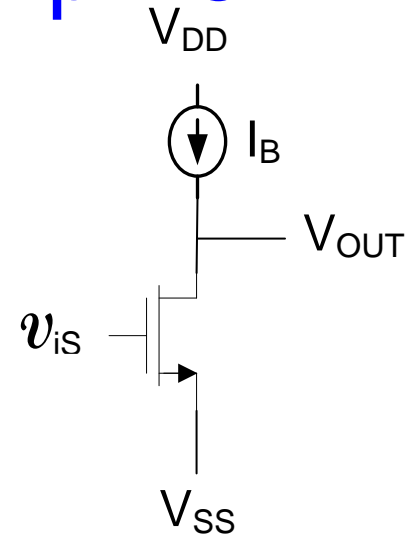
Linearity is reasonably good over practical input range

Practical input range is much less than  $V_{EB}$

# Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



$\lambda=0.01V^{-1}$  more realistic in many processes or for longer L

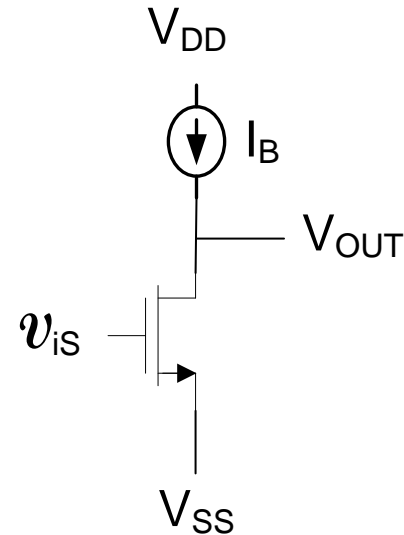
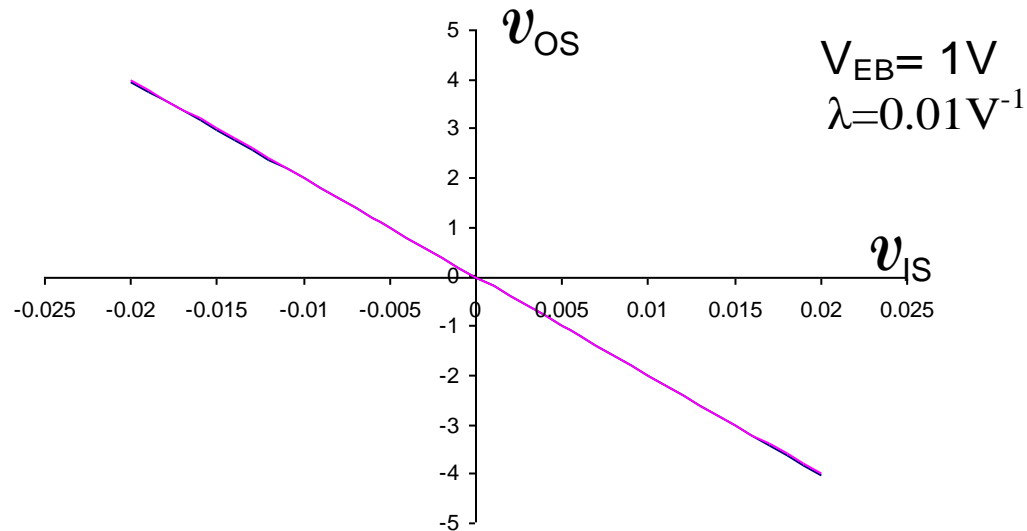
Can't "see" nonlinearity in this plot

Appears to be dependent upon dc gain of amplifier ??

# Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



Will look at difference between output and ideal output as defined by fit line

$$v_{FIT} \cong -\frac{2}{\lambda V_{EB}} v_{iS}$$

$$\varepsilon = v_{FIT} - v_{OS}$$

$$\varepsilon \cong \frac{1}{\lambda V_{EB}^2} v_{iS}^2$$

Appears to be highly dependent upon dc gain of amplifier ??

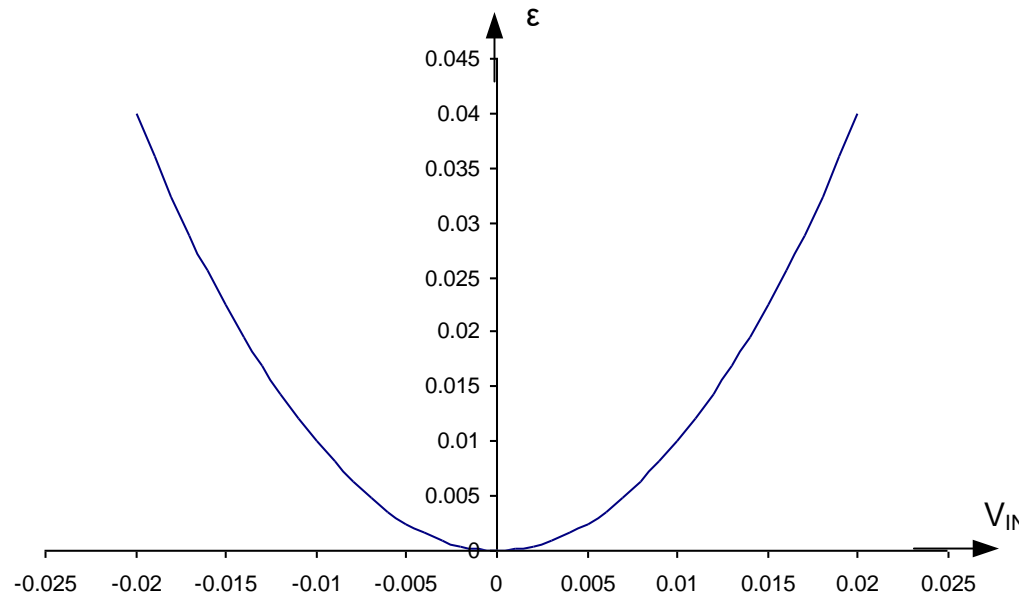
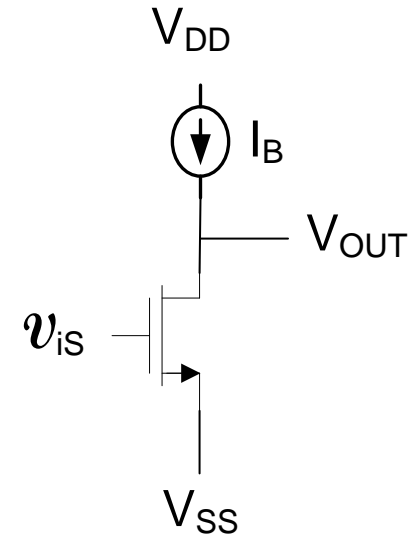
# Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?

$$\varepsilon \cong \frac{1}{\lambda V_{EB}^2} v_{iS}^2$$

$$V_{EB} = 1V$$
$$\lambda = 0.01V^{-1}$$

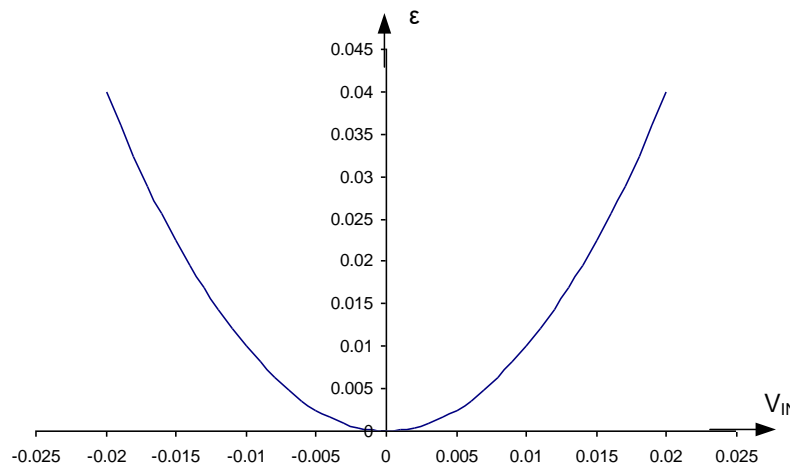
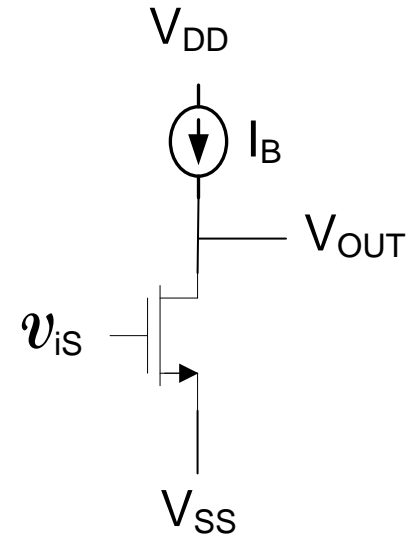


Appears to be highly dependent upon dc gain of amplifier ??

# Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



$$V_{EB} = 1V$$

$$\lambda = 0.01 V^{-1}$$

$$\epsilon_{PCT} \cong \frac{\epsilon}{v_{FIT}} 100\% = \left[ \frac{\frac{1}{\lambda V_{EB}^2} v_{iS}^2}{\frac{2v_{iS}}{\lambda V_{EB}}} \right] 100\% = \left( \frac{100\%}{2V_{EB}} \right) v_{iS}$$

$$\epsilon_{PCT} \cong \left( -\frac{\lambda \cdot 100\%}{4} \right) v_{OS}$$

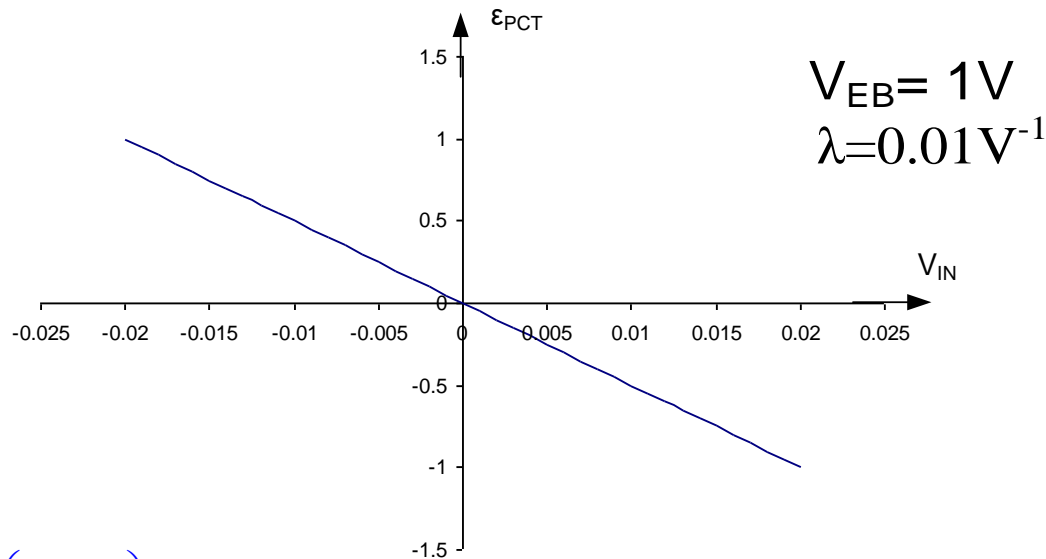
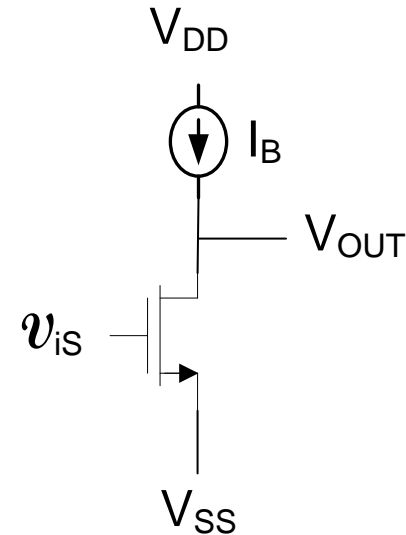
Appears to be highly dependent upon dc gain of amplifier ??

Relative error in output independent of gain of amplifier !

# Linearity of Common-Source Amplifier

$$v_{OS} \cong -\frac{2}{\lambda V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



$$\varepsilon_{PCT} \cong \left( \frac{100\%}{2V_{EB}} \right) v_{iS}$$

or, in terms of  $v_{OS}$ ,

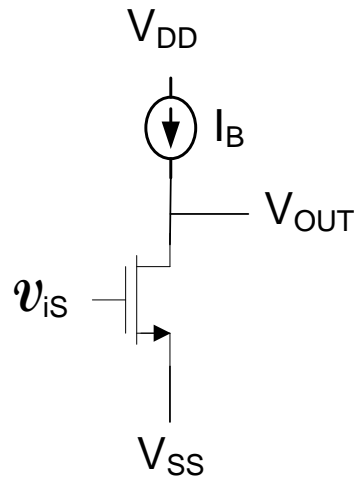
$$\varepsilon_{PCT} \cong \left( -\frac{\lambda \cdot 100\%}{4} \right) v_{OS}$$

1% deviation for this example occurs at

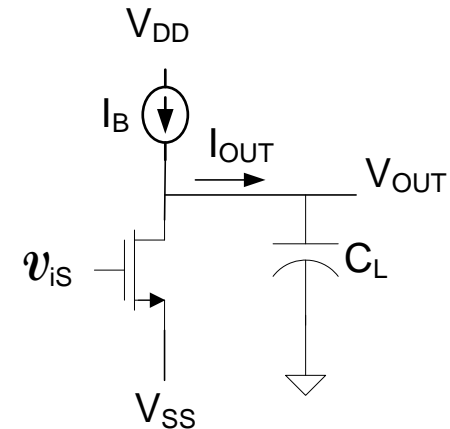
$$|v_{OS}| \cong 0.01 \frac{4}{\lambda} \cong 4V$$

In spite of square-law nonlinearity in MOSFET, linearity of CS amplifier is quite good provided MOSFET remains in saturation region !!

# Linearity of Common-Source Amplifier

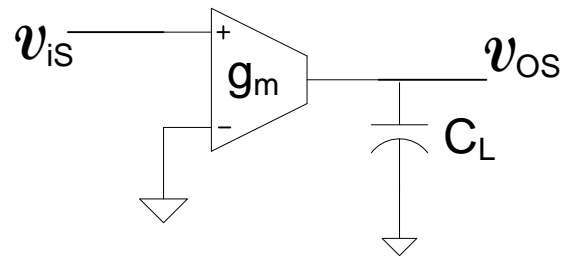


High-Gain Amplifier



Transconductance Amplifier

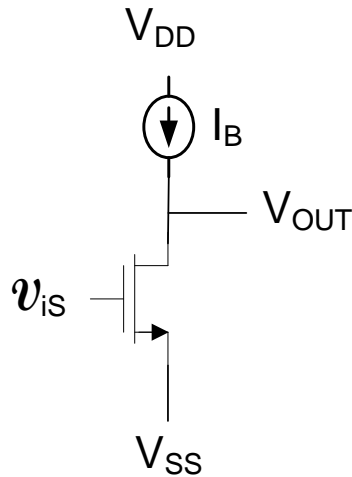
The transconductance amplifier driving a load  $C_L$  is performing as an integrator



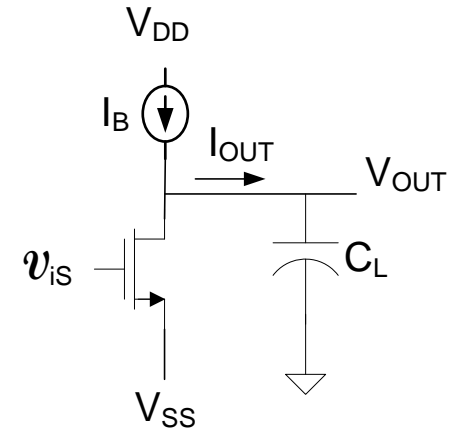
Integrators often used in filters where at frequencies of most interest  $|v_{OS}|$  is comparable to  $|v_{iS}|$

Is this common-source amplifier linear or nonlinear?

# Linearity of Common-Source Amplifier



High-Gain Amplifier



Transconductance Amplifier

$$I_{OUT} = I_B - I_D$$

$$I_{OUT} = I_B - \beta(v_{iS} + V_{EB})^2 (1 + \lambda[V_{OS} + V_{OQ} - V_{SS}])$$

$$I_{OUT} = \left[ I_B - \beta(V_{EB})^2 (1 + \lambda[V_{OQ} - V_{SS}]) \right] - \beta(v_{iS}^2 + 2v_{iS}V_{EB})(1 + \lambda[V_{OS} + V_{OQ} - V_{SS}])$$

$$I_{OUT} \cong -\beta(v_{iS}^2 + 2v_{iS}V_{EB})$$

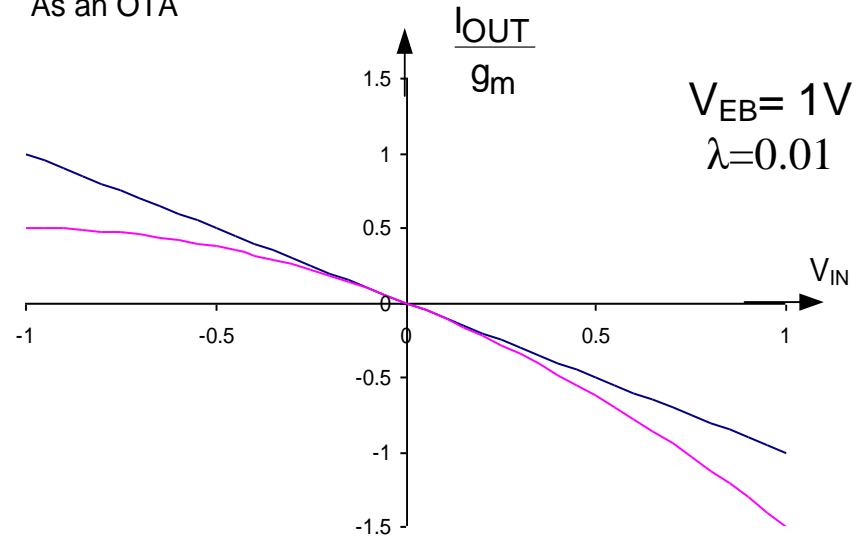
$$I_{OUT} \cong -\frac{2I_B}{V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$

Is this a linear or nonlinear relationship?



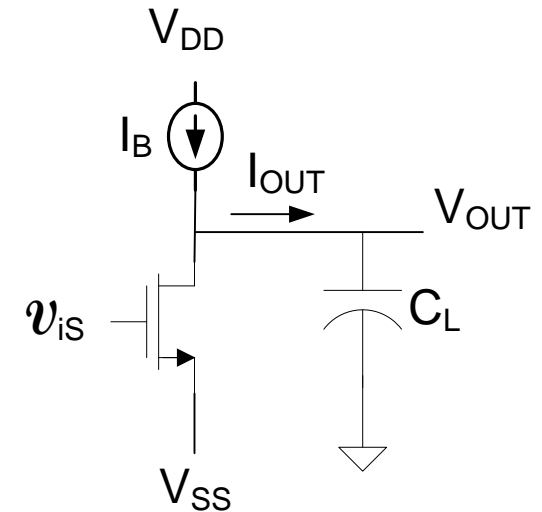
# Linearity of Common-Source Amplifier

As an OTA



$$g_m = \frac{2I_B}{V_{EB}}$$

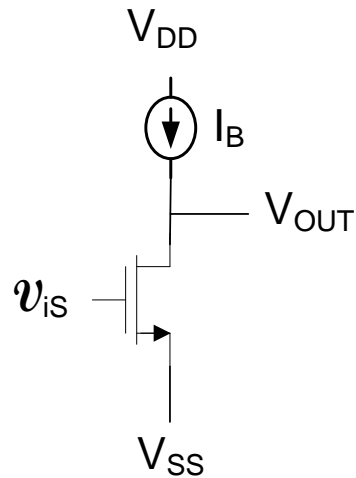
$$I_{OUT} \cong -\frac{2I_B}{V_{EB}} \left( v_{iS} + \frac{1}{2V_{EB}} v_{iS}^2 \right)$$



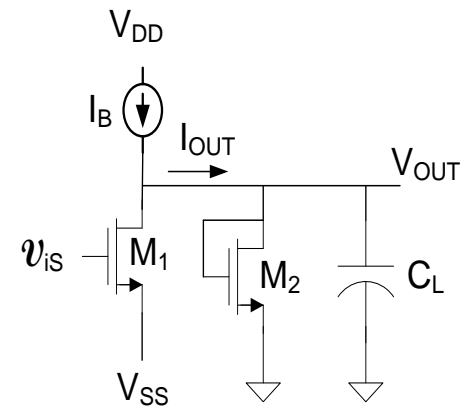
Is this a linear or nonlinear relationship?

At  $v_{iS} = -V_{EB}$ , the error in  $I_{OUT}$  will be -50% !

# Linearity of Common-Source Amplifier



High-Gain Amplifier

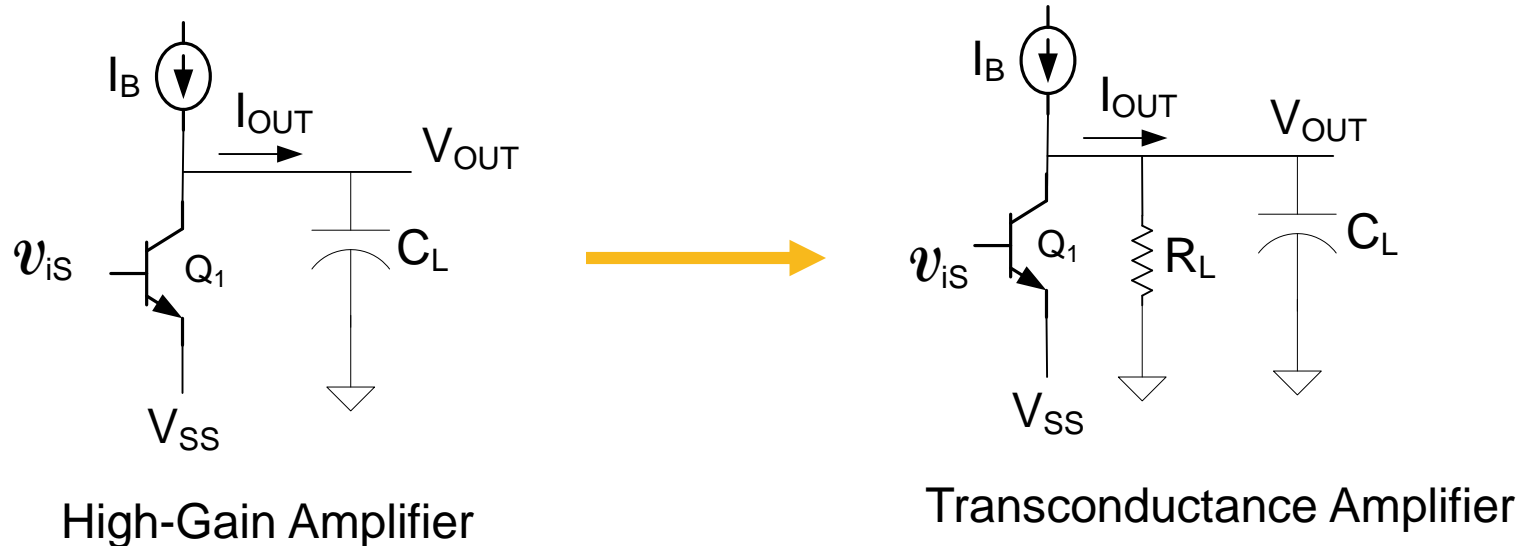


Transconductance Amplifier

## Is this common-source amplifier linear?

- Reasonably linear if used in high-gain applications and  $V_{EB}$  is large (e.g. if  $A_V = g_m/g_o = 2/((\lambda V_{EB})) = 100$  and  $V_o = 1V$ ,  $V_{in} = 10mV$ )
- Highly nonlinear when used in low-gain applications though linearity dependent upon  $g_m$

# Linearity of Common-Emitter Amplifier



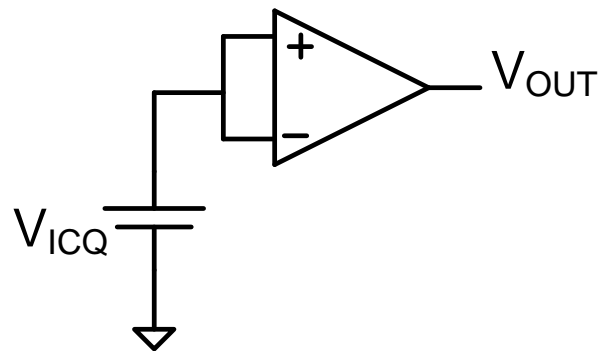
## Is this common-emitter amplifier linear?

- Very linear if used in high-gain applications  
(e.g. if  $A_V = g_m / g_0 = V_{AF} / V_t = 4000$  and  $V_o = 1V$ ,  $V_{in} = 250\mu V$ )
- Highly nonlinear when used in low-gain applications but not dependent upon  $g_m$
- Bipolar OTAs (e.g. current mirror op amp) can operate over multiple decades of gain with low-level signals but no degradation with gain

# Offset Voltage

Two types of offset voltage:

- Systematic Offset Voltage
- Random Offset Voltage

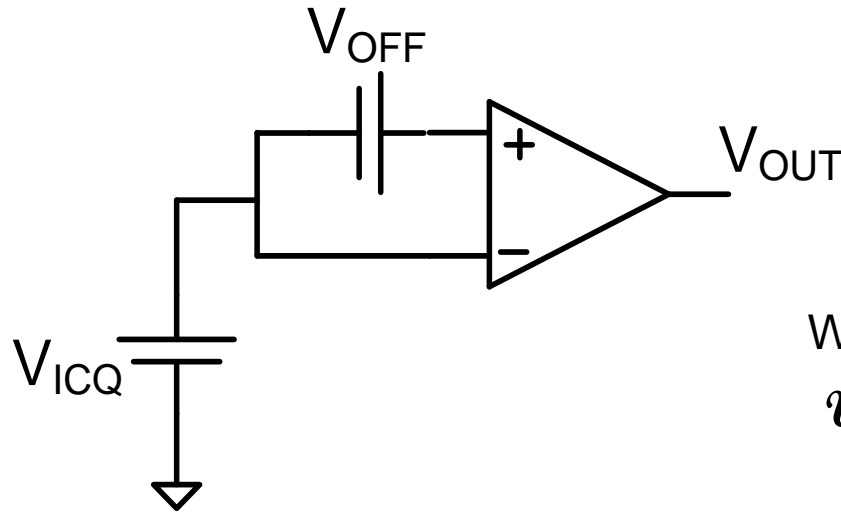


Definition: The output offset voltage is the difference between the desired output and the actual output when  $V_{id}=0$  and  $V_{ic}$  is the quiescent common-mode input voltage.

$$V_{OUTOFF} = V_{OUT} - V_{OUTDES}$$

Note:  $V_{OUTOFF}$  is dependent upon  $V_{ICQ}$  although this dependence is usually quite weak and often not specified

# Offset Voltage



When operating linearly  
 $v_{OUT} = A_D v_D + A_C v_C$

Definition: The input-referred offset voltage is the differential dc input voltage that must be applied to obtain the desired output when  $V_{ic}$  is the quiescent common-mode input voltage.

$V_{OFF}$  is usually related to the output offset voltage by the expression

$$V_{OFF} = \frac{V_{OUTOFF}}{A_D}$$

$V_{OFF}$  is dependent upon  $V_{ICQ}$  although this dependence is usually quite weak and often not specified

$V_{OFF}$  almost always large enough to force open-loop op amp out of linear mode for good op amps

# Offset Voltage

Note: Our definition differs from that of most others

From Wikipedia March 12, 2022

The **input offset voltage** ( $V_{os}$ ) is a parameter defining the differential DC voltage required between the inputs of an amplifier, especially an operational amplifier (op-amp), to make the output zero (for voltage amplifiers, 0 volts with respect to ground or between differential outputs, depending on the output type).<sup>[1]</sup>

From Analog Devices MT-037 Tutorial

Ideally, if both inputs of an op amp are at exactly the same voltage, then the output should be at zero volts. In practice, a small differential voltage must be applied to the inputs to force the output to zero. This is known as the input offset voltage,  $V_{OS}$

Offset Voltage: The differential voltage which must be applied to the input of an op amp to produce zero output.

# Offset Voltage

Note: Our definition differs from that of most others

From Texas Instruments Application Note: SLOA059 – March 2001

## Input Offset Voltage Defined

The input offset voltage is defined as the voltage that must be applied between the two input terminals of the op amp to obtain zero volts at the output. Ideally the output of the op amp should be at zero volts when the inputs are grounded.

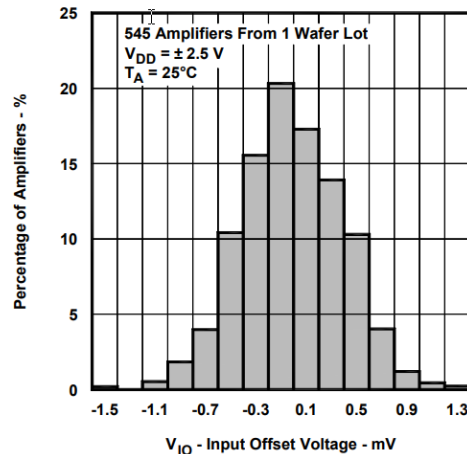


Figure 3. Distribution of  $V_{IO}$  for the TLV2721

Implicit in the definition of  $V_{OS}$  by most others is that the desired output voltage of an op amp is 0V when the differential input is 0V

# Offset Voltage

From Analog Devices MT-037 Tutorial

## ◆ Ranges:

- Chopper Stabilized Op Amps:  $<1\mu\text{V}$
- General Purpose Precision Op Amps:  $50\text{-}500\mu\text{V}$
- Best Bipolar Op Amps:  $10\text{-}25\mu\text{V}$
- Best JFET Input Op Amps:  $100\text{-}1,000\mu\text{V}$
- High Speed Op Amps:  $100\text{-}2,000\mu\text{V}$
- Untrimmed CMOS Op Amps:  $5,000\text{-}50,000\mu\text{V}$
- DigiTrim™ CMOS Op Amps:  $<100\mu\text{V}\text{-}1,000\mu\text{V}$

*Figure 1: Typical Op Amp Input Offset Voltage*

These ranges probably are applicable to catalog op amps



# Offset Voltage

From Analog Devices MT-037 Tutorial

## INPUT OFFSET VOLTAGE DRIFT AND AGING EFFECTS

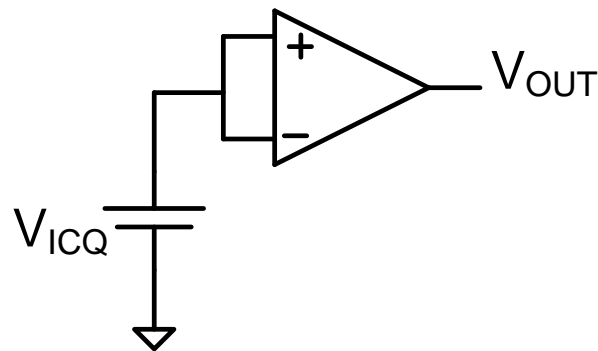
**Input offset voltage varies with temperature**, and its temperature coefficient is known as  $TCV_{os}$ , or more commonly, *drift*. Offset drift is affected by offset adjustments to the op amp, but when the offset voltage of a bipolar input op amp has been minimized, the drift may be as low as  $0.1 \mu\text{V}/^\circ\text{C}$  (typical value for [OP177F](#)). More typical drift values for a range of general purpose precision op amps lie in the range  $1\text{-}10 \mu\text{V}/^\circ\text{C}$ . Most op amps have a specified value of  $TCV_{os}$ , but some, instead, have a second value of maximum  $V_{os}$  that is guaranteed over the operating temperature range. Such a specification is less useful, because there is no guarantee that  $TCV_{os}$  is constant or monotonic.

**The offset voltage also changes as time passes, or ages**. Aging is generally specified in  $\mu\text{V}/\text{month}$  or  $\mu\text{V}/1000 \text{ hours}$ , but this can be misleading. Since aging is a "drunkard's walk" phenomenon, it is proportional to the *square root* of the elapsed time. An aging rate of  $1 \mu\text{V}/1000 \text{ hour}$  therefore becomes about  $3 \mu\text{V}/\text{year}$  (not  $9 \mu\text{V}/\text{year}$ ).

# Offset Voltage

Two types of offset voltage:

- Systematic Offset Voltage
- Random Offset Voltage



After fabrication it is impossible (difficult) to distinguish between the systematic offset and the random offset in any individual op amp

Measurements of offset voltages for a large number of devices will provide mechanism for identifying systematic offset and statistical characteristics of the random offset voltage

# Systematic Offset Voltage

Offset voltage that is present if all device and model parameters assume their nominal value

Easy to simulate the systematic offset voltage

Almost always the designer's responsibility to make systematic offset voltage very small

Generally easy to make the systematic offset voltage small

# Random Offset Voltage

- Due to random variations in process parameters and device dimensions
- Random offset is actually a random variable at the design level but deterministic after fabrication in any specific device
- Distribution naturally nearly Gaussian (could be un-naturally manipulated)

Has zero mean

Characterized by its standard deviation or variance

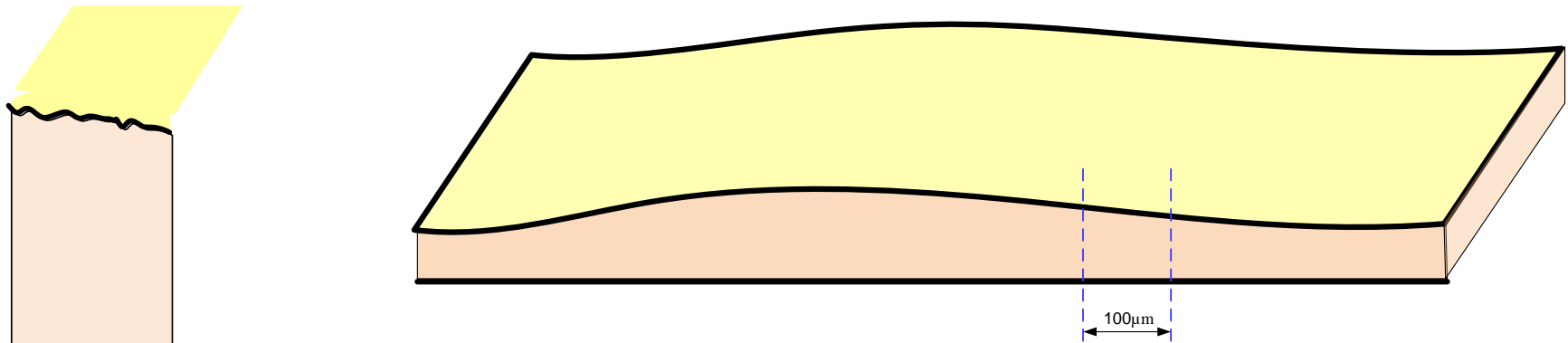
Often strongly layout dependent

Due to both local random variations and correlated gradient effects

- Will consider both effects separately
- Gradient effects usually dominate if not managed
- Good methods exist for driving gradient effects to small levels

# Gradient and Local Random Effect

1



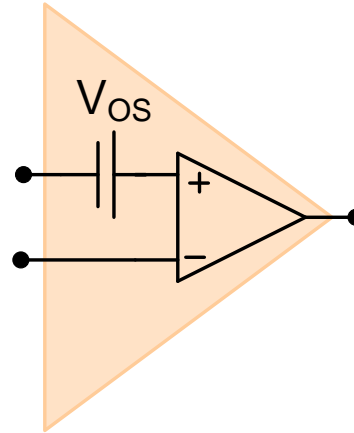
Gradient Effects : Locally Appear Linear

- Magnitude and Direction of Gradients are random
- Highly Correlated over Short Distances

Local Random Effects :  
Vary Locally With No  
Correlation

- **Both Contribute to Offset**
- **Both are random variables**
- **If Not Managed, Both Can Cause Large Offsets**
- **Strategies for minimizing their effects are different**
- **Will refer to the local random effects as “random” and the random gradient effects as “gradient” effects**

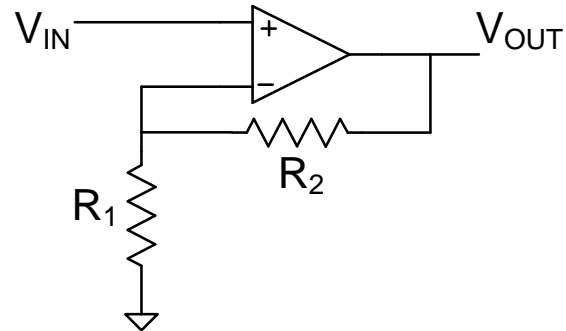
# Offset Voltage



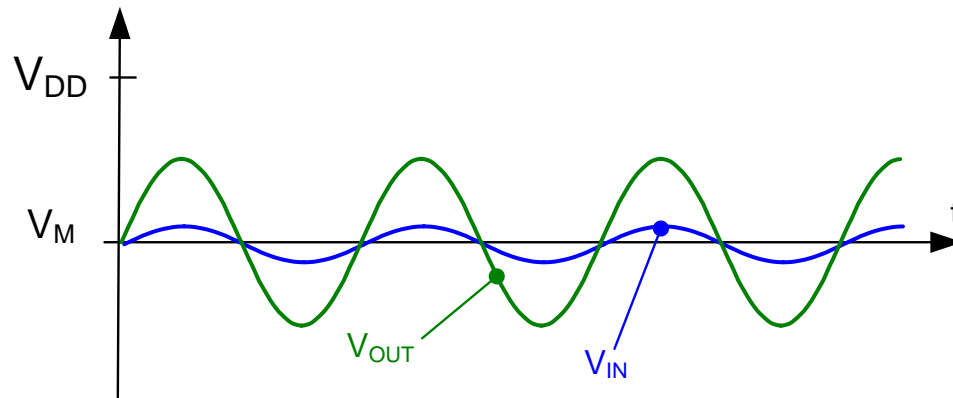
Can be modeled as a dc voltage source in series with the input

# Offset Voltage

Effects of Offset Voltage - an example



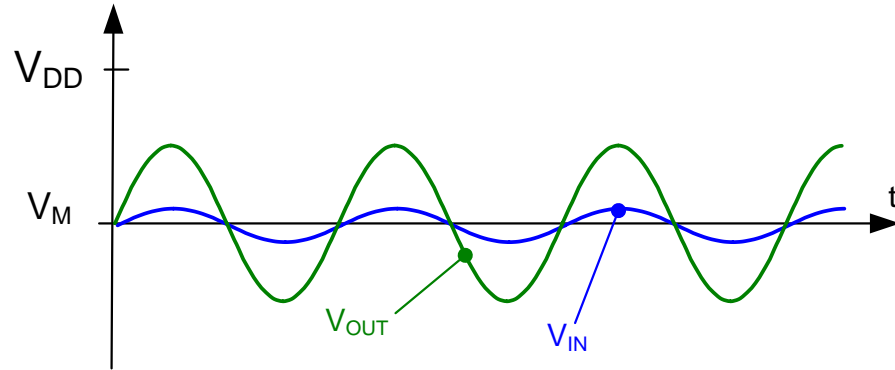
Desired I/O relationship



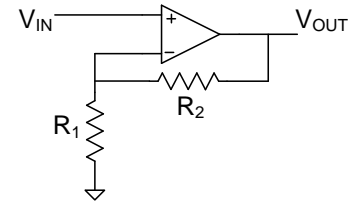
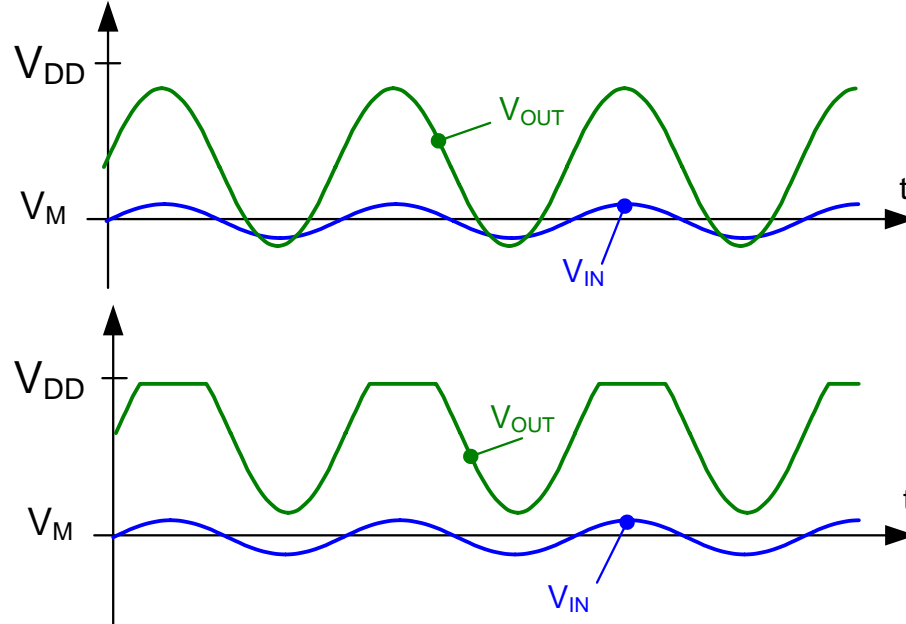
# Offset Voltage

Effects of Offset Voltage - an example

Desired I/O relationship

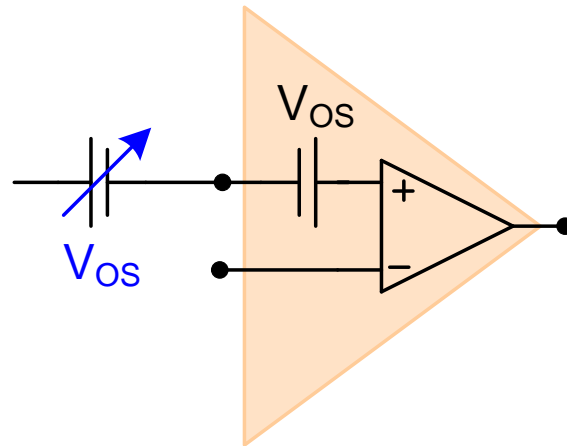


Actual I/O relationship due to offset





# Offset Voltage



Effects can be reduced or eliminated by adding equal amplitude opposite DC signal (many ways to do this)

Widely used in offset-critical applications

Comes at considerable effort and expense for low offset

**Prefer to have designer make  $V_{os}$  small in the first place**

# Effects of Offset Voltage

- Deviations in performance will change from one instantiation to another due to the random component of the offset
- Particularly problematic in high-gain circuits
- A major problem in many other applications
- Not of concern in many applications as well



Stay Safe and Stay Healthy !

End of Lecture 21